2

Answers and Hints

2.8 Second Price Auction

Suppose first you win, and let v_s be the second-highest bid. If you had bid more than v_i, you still would have won, and your winning would still be the same, namely $v_i - v_s \geq 0$. If you had bid lower that v_i, there are three subcases: you could have bid more than, equal to, or less than v_s. If you had bid more than v_s, you would have had the same payoff, $v_i - v_s$. If you had bid equal to v_s, you could have lost the auction in the playoff among the equally-high bidders, and if you had bid less than v_s you certainly would have lost the auction. Hence nothing beats bidding v_i in case you win.

But suppose you bid v_i and lost, and let v_h be the highest bid and v_s be the second-highest bid. Since you lost, your payoff is zero, so if you had bid less than v_i, you would still have lost the auction, so you couldn't improve your payoff this way. If had you bid more than v_i, it wouldn't matter unless you had bid enough to win the auction, in which case your gain would have been $v_s - v_i$. Since $v_i \neq v_h$, we must have $v_i \leq v_s$, since v_s is the second-highest offer. Thus you could not have made a positive gain by bidding higher than v_i.

Hence bidding v_i is a best response to any set of bids by the other players.

2.10 Hagar's Battles

battlefields. To see this suppose player 1 does not. Let x_j be the highest value battlefield unoccupied by player 1, and let x_i be the lowest value battlefield occupied by player 1. What does player 1 gain by switching a soldier from x_i to x_j? If both or neither are occupied by player 2, there is no change. If player 2 occupies x_j but not x_i, player 1 loses a_i by switching, and player 2 loses a_j, so player 1 gains a_j - $a_i > 0$. Similarly

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if player 2 occupies x_i but not x_j.

Here is another explanation: Suppose you occupy a_i but not a_j, where $a_j > a_i$. Consider the effect of switching from a_i to a_j:

	Enemy Occupies			
	a_i not a_j	a_j not a_i	a_i and a_j	neither
loss	lose i	lose i	lose i	lose i
gain	gain j	gain j	gain j	gain j
net gain	$j - i$	$j - i$	$j - i$	$j - i$

Clearly the gain from switching is positive in all contingencies.

2.13 A Military Strategy Game

at A. This leaves six strategies, which we can label fcb, feb, fed, hed, heb, and hgd. We can also eliminate all Country A strategies that stay at A at any time, or that hit h or f. This leaves the six strategies bcb,beb,bed,ded,deb,dgd. Here is the payoff matrix:

	bcb	beb	bed	ded	deb	dgd
fcb	-1	-1	1	1	-1	1
feb	-1	-1	1	-1	-1	1
fed	1	-1	-1	-1	-1	-1
hed	1	-1	-1	-1	-1	-1
heb	-1	-1	-1	-1	-1	1
hgd	1	1	-1	-1	1	-1

Now feb is weakly dominated by fcb, as is heb. Moreover we see that fed and hed are weakly dominated by hgd. Thus there are two remaining strategies for Country I, 'south' (hgd) and 'north' (fcb).

Also bcb is dominated by beb and dgd is dominated by ded, so we may drop them. Moreover, beb and deb are the same 'patrol north', while bed and ded are the same 'patrol south.' This gives us the following reduced game:

	patrol north	patrol south
attack north	-1,1	1,-1
attack south	1,-1	-1,1

So this complicated game is just the heads-tails game, which we will finish solving when we do mixed strategy equilibria!

2.14 Strategic Voting

by pruning the game tree has player 1 choosing no, and players 2 and 3 choosing yes, with payoff $(b, b-c, b-c)$. It is best to go first.

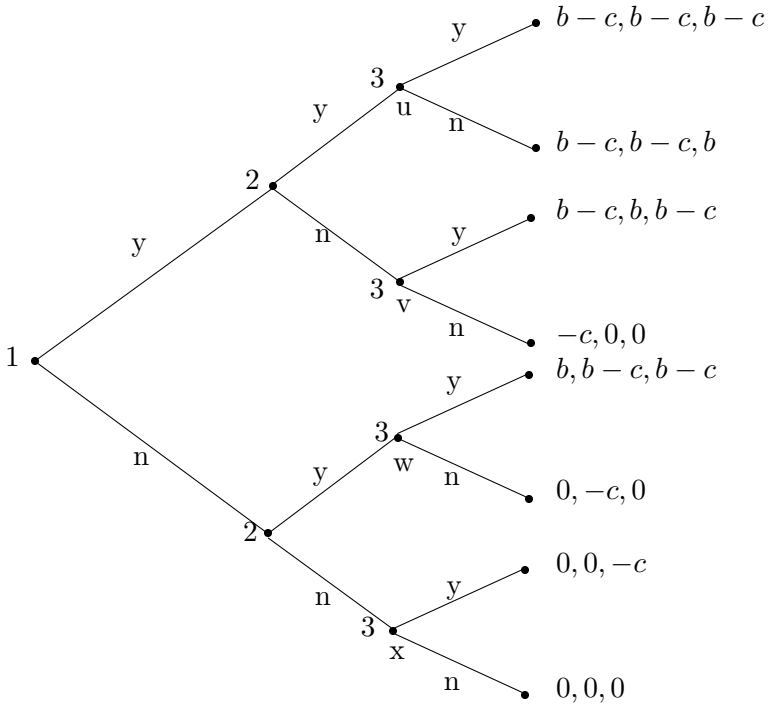

Figure 2.1: A Game Tree for Strategic Voting

Notice that this does not give a full specification of the strategies, since player 2 has four strategies and player 3 has sixteen strategies. The above description says only what players 2 and 3 do "along the game path"; i.e., as the game is actually played.

To describe the Nash equilibrium in full, let us write player 3's strategies as 'uvwx', where u, v, w, and x are each either y ('yes') or n ('no'),

and indicate the choice at the corresponding node in the game tree, starting from the top. Then the third player's choice is nyyn. Similarly, player 2's choice is ny, and the first's is, of course, n.

If player 3 chooses nnnn player 2 chooses yn, and player 1 chooses y, we have another Nash equilibrium (check it out!), in which player 3 now gets b and the other two get $b - c$. The equilibrium is strange because it means that player 3 should make a suboptimal choice at node u—he says he will choose 'no' but in fact he will choose 'yes' at u, since it gives him a higher payoff. The strategy nnnn is called an *incredible threat*, because it involves player 3 threatening to do something that he in fact will not do when it comes time to do it. But if the other believe him, he will never have to carry out his threat! We say such a Nash equilibrium violates *subgame perfection*, a phenomenon of great importance that we have touched upon in the Big Monkey and Little Monkey game 1.2, and we will take up systematically in Chapter 5.